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Hybrid Efficient Control Algorithms for Robot Manipulators

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ABSTRACT

In this report, we discuss accurate and robust sliding mode tracking control for highly nonlinear robot manipulators using a disturbance observer. To eliminate the chattering problem existed in conventional Sliding Mode Control (SMC) approach, which is caused by modeling errors and uncertainties. The efficient compensation of the disturbance observer has been introduced.

The proposed sliding mode control is presented in two theorems. The bounded stability of the proposed control method is proven and the efficiency of the control algorithms has been demonstrated by simulations for a position tracking control of a two-link robot subject to parameter and payload uncertainties.

1 Introduction

The overall robot control problem can be divided into two phases, trajectory planing and control phases. The trajectory planing is the planing of the desired time trajectory from an initial position to a desired position of robot manipulators with collision avoidance if necessary. The control phase is the tracking control of robot manipulators to track the desired trajectory planned in the trajectory planning phase. This tracking control phase is the field of our concern in this paper. The strategies for designing a robust and accurate tracking controller for a highly nonlinear robot manipulator have been studied enthusiastically in order to extend its application fields. There are several approaches to attempt to obtain the desired tracking performance such as, Computed Torque Method (CTM) [1-3], Adaptive Control [4, 5], Sliding Mode Control (SMC) [6-13], and so on [14]. Each method has its merits and shortcomings. The CTM normally provides a feasible controller if the exact knowledge of the manipulator dynamics is available. However, for a large amount of applications, it is impossible to obtain the complete dynamic model of robots, due to modeling uncertainties, parameter variation and unknown payloads. These uncertainties, especially the error of inertia matrix, may result in the instability of robot systems. All of highly nonlinear model dynamics are taken into account in order to calculate the control input which is a hard load in view of the computation time. Thus the optimal trade-off is made between approximate modeling of the robot manipulator and its output performance. The CTM is sensitive to modeling errors which come from linearization and approximation of nonlinearities, uncertainties in physical systems, and disturbances [3].

In order to give the robustness to modeling uncertainties and parameter variations, adaptive algorithms for a highly nonlinear robot manipulator are considered such as Self-Tuning and Model-Reference Methods but essentially need an adaptation mechanism for the parameter identification resulting in a heavy computation burden, complexities, and a high cost for the digital implementation. As another robust controller different from the adaptive approach, SMC using Variable Structure Systems (VSS) for robot arms is studied by many researchers recently [6-13]. The desired performance with a simple control structure can be obtained in the existence of the acceptable model error and unknown payload using the "*sliding mode*". In the sliding mode, it is well known that the controlled system is robust to the bounded parameter variations and disturbances [17, 18]. The other advantages of a SMC are that the output performance can be predetermined by a *sliding surface* and there is no overshoot in regulations. The first application of SMC to robot manipulators seems to be in the work of Young dealing with a set point regulation problem [6]. A modification of the Young controller was presented by Morgan [8]. Other SMC of robot manipulator may be found [10-12]. The SMC unfortunately has a problem that the motion trajectory of system changes frequently in the vicinity of the sliding surface due to a high frequency switching of the input which is called as "*chattering*". Because in order to guarantee the existence of the sliding mode, the control has imperatively the discontinuities designed by the maximum bound of model errors or uncertainties. The conventional SMCs for robot manipulators use the feedforward compensation by using an available model dynamics, there exist inevitable chattering due to model errors or uncertainties in system parameters. To alleviate this problem bounded layer method or saturation function [9] are used instead of discontinuous

parts. These methods are based on the try-and-error not a systematic approach and not enough to obtain the desired performance with the satisfactory tracking error. Thus as possible as the exact compensation of the high nonlinear interactions in robot link systems is essential to obtain the good performance without chattering. Using the multirate sampling concept, a SMC with an off-line feedforward compensation for the nonlinear interaction of robot manipulators using an available model is presented.

In this paper, design of a sliding mode control algorithm with an efficient on-line compensation for robot manipulators is studied for robust and accurate tracking of the desired trajectories. The proposed control consists of two parts of the efficient compensation and the sliding mode only based on the nominal inertia matrix of robot manipulators. This compensation technique is responsible for not only highly nonlinear interactions but also modeling errors of the inertia matrix using the acceleration information and the nominal inertia matrix. The sliding mode control is applied to the compensated dynamics of robot manipulators to obtain robust and accurate tracking performance with the correction of compensation errors in view of the real implementation using the microprocessor. The stability of the proposed SMC is deeply analyzed in sense of Lyapunov, summarized in two main theorems with a preliminary lemma. Based on this analysis, the given desired tracking specifications can be satisfied by a proposed SMC. The design of the proposed SMC is independent of the uncertainty parameter space. Thus there can be no chattering due to modeling errors or uncertainties.

The rest of the paper is organized as follows. Section 2 gives the mathematical basis for a stable SMC design by Lemma 1. In Section 2, a new SMC with the efficient compensation is proposed, and its stability is analyzed in Theorem 1. A modified control is then presented in Section 3 and another more efficient approach is proposed in Section 4. Its stability is summarized in Theorem 2. The simulation studies for the proposed algorithms are carried out in Section 5 and conclusions are presented in Section 6.

2 Mathematical Work

The motion equations of an n degree-of-freedom manipulator can be derived using the Lagrange-Euler formulation as

$$J(q(t), \phi) \cdot \ddot{q}(t) + D(q(t), \dot{q}(t), \phi) = \tau(t) \quad (1)$$

where $J(q(t), \phi) \in R^{n \times n}$ is a symmetric positive definite inertia matrix. $D(q(t), \dot{q}(t), \phi) \in R^n$ is called a *smooth generalized disturbances* (SGD) vector including the centrifugal and Coriolis terms $H(q(t), \dot{q}(t), \phi) \in R^n$, Coulomb and viscous or any other frictions $F(q(t), \dot{q}(t), \phi) \in R^n$, gravity terms $G(q(t), \phi) \in R^n$.

$$D(q(t), \dot{q}(t), \phi) = H(q(t), \dot{q}(t), \phi) + F(q(t), \dot{q}(t), \phi) + G(q(t), \phi)$$

where $\tau(t) \in R^n$ is an input vector, $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t) \in R^n$ are the generalized position, velocity, and acceleration vector, respectively, ϕ is a vector composed of the parameters of robot manipulators (i.e. the masses, lengths, offset angles, and inertia of links). Exact modeling of robot dynamics is difficult because of parameter uncertainties, unknown frictions.

payloads variations. Therefore, by using estimated parameters in the model, we express the model of the robot system (1) as follows

$$\hat{J}[q(t), \hat{\phi}] \cdot \ddot{q}(t) + \hat{D}(q(t), \dot{q}(t), \hat{\phi}) = \tau(t). \quad (2)$$

The objective of robot trajectory control is to follow the desired trajectories $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ computed in planning phase. Let's define the state vector $X(t) \in R^{2n}$ for SMC as

$$X(t) \equiv [X_1(t)^T \ X_2(t)^T]^T \quad (3)$$

where $X_1(t)$ and $X_2(t) \in R^n$ are the trajectory errors and its derivative

$$\begin{aligned} X_1(t) &\equiv e(t) = (q_d(t) - q(t)) \\ X_2(t) &\equiv \dot{e}(t) = (\dot{q}_d(t) - \dot{q}(t)). \end{aligned} \quad (4)$$

Then the state equation of the robot system becomes

$$\dot{X}(t) = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \cdot X(t) - \begin{bmatrix} 0 \\ J(q(t), \phi)^{-1} \end{bmatrix} \cdot \tau(t) + \begin{bmatrix} 0 \\ J(q(t), \phi)^{-1} \cdot D(q(t), \dot{q}(t), \phi) \end{bmatrix} + \begin{bmatrix} 0 \\ \ddot{q}_d(t) \end{bmatrix}. \quad (5)$$

An augmented sliding surface vector $s(t)$ is defined as

$$s(t) \equiv X_2(t) + K_\nu \cdot X_1(t) + K_\rho \cdot \int X_1(t) dt \quad (6)$$

where K_ν and $K_\rho \in R^{n \times n}$ are gain matrices. The ideal sliding dynamics defined by Equation (5) is the trajectories q^* , \dot{q}^* , $\ddot{q}^* \in R^n$ which satisfy the following equation

$$\ddot{X}_2^*(t) + K_\nu \cdot \dot{X}_2^*(t) + K_\rho \cdot X_1^*(t) = 0. \quad (7)$$

Or

$$\dot{X}^*(t) = \Lambda \cdot X^*(t) \quad (8)$$

where

$$X^*(t) = \begin{bmatrix} (q_d(t) - q^*(t))^T \\ (\dot{q}_d(t) - \dot{q}^*(t))^T \end{bmatrix}^T \in R^{2n}, \quad \Lambda = \begin{bmatrix} 0 & I \\ -K_\rho & -K_\nu \end{bmatrix} \in R^{2n \times 2n}.$$

We can choose K_ν and K_ρ so that all the eigenvalues of Λ have negative real parts, which guarantees the exponential stability of the system (7), i.e., there exist positive constants K and κ such that

$$\|e^{\Lambda t}\| \leq K \cdot e^{-\kappa t} \quad (9)$$

where $\|\cdot\|$ is the induced Euclidean norm [15].

Now, we will state the following lemma as a prerequisite to the two main theorems.

Lemma 1 : If the sliding surface defined by Equation (6) satisfies $\|s(t)\| \leq \gamma$ for any $t \geq t_0$ and $\|X(t_0)\| \leq \gamma/\kappa$ is satisfied at the initial time, then

$$\|X_1(t)\| \leq \varepsilon_1 \quad (10a)$$

$$\|X_2(t)\| \leq \varepsilon_2 \quad (10b)$$

is satisfied for all $t \geq t_0$ where ε_1 and ε_2 are positive constants defined as following

$$\varepsilon_1 = \frac{K}{\kappa} \cdot \gamma, \quad \varepsilon_2 = \gamma \cdot [1 + Z \cdot \frac{K}{\kappa}], \quad Z \equiv \| [K_\rho K_\nu] \| . \quad (10c)$$

Proof [16]

Define a new state, $X_0(t)$, to be an integral of the state $X_1(t)$, i.e.,

$$X_0(t) = \int_{t_0}^t X_1(u) du, \quad X_0(t_0) = 0. \quad (11)$$

then $X_0(t)$, the augmented sliding surface (6) can be rewritten as

$$s(t) = X_2(t) + K_\nu \cdot X_1(t) + K_\rho \cdot X_0(t). \quad (12)$$

If we define a new state vector $\bar{X}(t) = [X_0(t)^T, X_1(t)^T]^T$, thus we have the following state-space equation

$$\dot{\bar{X}}(t) = \Lambda \cdot \bar{X}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(t). \quad (13)$$

Where $s(t)$ may be considered as bounded disturbances, $\|s(t)\| \leq \gamma$. The solution of Equation (13) is

$$\bar{X}(t) = e^{\Lambda(t-t_0)} \cdot \bar{X}(t_0) + \int_{t_0}^t [e^{\Lambda(t-u)} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(u)] du. \quad (14)$$

From Equation (9) and boundedness of $s(t)$, the norm of $\bar{X}(t)$ becomes

$$\begin{aligned} \|\bar{X}(t)\| &\leq K \cdot e^{-\kappa(t-t_0)} \cdot \|\bar{X}(t_0)\| + \int_{t_0}^t \|e^{\Lambda(t-u)}\| du \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot \gamma \\ \|\bar{X}(t)\| &\leq \frac{K}{\kappa} \cdot \gamma + [\|\bar{X}(t_0)\| - \frac{\gamma}{\kappa}] \cdot K \cdot e^{-\kappa(t-t_0)} \\ &\leq \gamma \cdot \frac{K}{\kappa} \quad \text{for all } t \geq t_0, \end{aligned} \quad (15)$$

which drives $X_1(t)$ to satisfy the following inequality

$$\|X_1(t)\| \leq \frac{K}{\kappa} \cdot \gamma.$$

From Equation (12) we have

$$X_2(t) = s(t) - [K_\rho K_\nu] \cdot \bar{X}(t). \quad (16)$$

then by using the result of Equation (15)

$$\|X_2(t)\| \leq \gamma \cdot [1 + Z \cdot \frac{K}{\kappa}]$$

for all $t \geq t_0$ which proves (10b) and therefore we complete our proof.

Lemma 1 implies that the tracking error of the trajectory and its derivative are uniformly bounded, provided that the sliding surface is bounded for all time $t \geq t_0$. Using this results of Lemma 1, we can give the specifications of tracking errors dependent on convergence rate which is determined by the sliding surface, (9). In the next section, we are going to design a controller with efficient compensation which guarantees the boundedness of $s(t)$, i.e., if $\|s(t)\| \leq \gamma$ for a given γ , then the trajectory error is bounded in virtue of Lemma 1.

3 Efficient Sliding Mode Control for Robot Manipulators

There are a variety of disturbances acting on robot manipulators, which make robust tracking control highly desirable. It is commonly noted that the generalized nonlinear disturbances, $D(q(t), \dot{q}(t), \phi)$, should be compensated to improve performance. For sliding mode control of the robot system (4), the equivalent control of the augmented sliding surface (5) is obtained [17, 18]

$$\tau_{eq}(t) = D(q(t), \dot{q}(t), \phi) + J(q(t), \phi) \cdot (\ddot{q}_d(t) + K_v X_2 + K_p X_1) \quad (17)$$

The SGD $D(q(t), \dot{q}(t), \phi)$ is included in an equivalent control, $\tau_{eq}(t)$. SGD is generally complex, and difficult to be calculated directly due to uncertainties in the model.

In the paper, using the efficient estimation method so called *disturbance observer*, we consider the following continuous control input, $\tau(t)$

$$\tau(t) = \tau_c(t) + \tau_s(t) \quad (18)$$

Where $\tau_c(t)$ is the compensation term for SGD as well as the error of nominal inertia matrix. is not the direct calculation from $\hat{D}(q(t), \dot{q}(t), \hat{\phi})$ in the model (2) but the efficient estimation of the generalized disturbance, $D(q(t), \dot{q}(t), \phi)$ only using the nominal inertia matrix, J_N , of the model (2) and an available acceleration information which can be obtained from the speed information by means of Euler method.

$$\begin{aligned} \tau_c(t) &= \tau(t) - J_N \cdot \ddot{q}(t) \\ &= D(q(t), \dot{q}(t), \phi) + \Delta J(q(t), \phi) \cdot \ddot{q}(t) - J(q(t), \phi) \cdot \Delta \ddot{q}(t) - \Delta \tau(t) \end{aligned} \quad (19)$$

where $\Delta J(q(t), \phi)$ is the deviation between the real inertia matrix and its nominal value, $\Delta \ddot{q}$ is the acceleration information error to the real acceleration value, $\Delta \tau(t)$ is the control input delay error resulting from the digital control are defined by

$$\Delta J(q(t), \phi) = J(q(t), \phi) - J_N$$

$$\Delta \ddot{q}(t) = \ddot{q}(t) - \ddot{q}(t)$$

$$\Delta \tau(t) = \tau(t - h) - \tau(t)$$

where h is the sampling time. The second term in Equation (16) is defined as

$$\tau_s(t) = (\bar{\tau}_{eq}(t) + \tau_x(t)) \quad (20)$$

where $\bar{\tau}_{eq}(t)$ is the modified equivalent control for the compensated dynamics of Equation (1), and is designed so that the error dynamics of the controlled system has the sliding surface dynamics defined by Equation (7). It is defined as

$$\bar{\tau}_{eq}(t) = J_N \cdot (\ddot{q}_d(t) + K_v \cdot X_2 + K_p \cdot X_1). \quad (21)$$

$\tau_x(t)$ is the feedback term of the sliding surface resulting from the compensation error

$$\tau_x(t) = J_N \cdot \{k_{x1} \cdot s(t) + k_{x2} \cdot \sigma_a(t)\}, \quad \sigma_a(t) = \frac{s(t)}{\|s(t)\| + \delta}. \quad (22)$$

If we apply the input control torque given by Equations (18)-(22) to the robot system (5), we obtain the following equation

$$\begin{aligned} \dot{X}_2(t) = & -J^{-1}(q(t), \phi) \cdot (\Delta J(q(t), \phi) \cdot \ddot{q}(t) - \Delta \tau(t)) + \Delta \ddot{q}(t) + \ddot{q}_d \\ & - J^{-1}(q(t), \phi) \cdot J_N \cdot [\ddot{q}_d + K_v \cdot X_2 + K_p \cdot X_1 + k_{x1} \cdot s(t) + k_{x2} \cdot \frac{s(t)}{\|s(t)\| + \delta}] \end{aligned} \quad (23)$$

and the dynamics of $s(t)$ is expressed in the following simple form

$$\dot{s}(t) = n_1(t) - [k_{x1} \cdot s(t) + k_{x2} \cdot \frac{s(t)}{\|s(t)\| + \delta}] \quad (24)$$

where $n_1(t) \in R^n$ is the resultant disturbance vector given by

$$\begin{aligned} n_1(t) &= n_1(\Delta \ddot{q}(t), \Delta \tau, \hat{\phi}) \\ &= J_N^{-1} \cdot J(q(t), \phi) \cdot \Delta \ddot{q} + J_N^{-1} \cdot \Delta \tau(t). \end{aligned} \quad (25)$$

For some positive constants ε_1 and ε_2 defined in (10c), let the constant N be defined as follows

$$N = \max_{t, q, \dot{q}, \phi} \{\|n_1(\Delta \ddot{q}(t), \Delta \tau(t), \hat{\phi})\|; q(t) \in B(\varepsilon_1; q_d(t)) \wedge \dot{q}(t) \in B(\varepsilon_2; \dot{q}_d(t))\} \quad (26)$$

where the matrix norm is defined as the induced Euclidean norm, and for a positive number $\rho > 0$ and a vector $\nu \in R^n$ the boundary set defined by as

$$B(\rho; \nu) = \{w \in R^n; \|w - \nu\| \leq \rho\}. \quad (27)$$

In Equation (25), the resultant disturbances are mainly dependent on the acceleration information error and the control input computation delay error other than system uncertainties or modeling errors of robot manipulators. The disturbance observer can compensate modeling errors of the inertia matrix besides SGD. Thus the SMC design is independent of maximum bound of modeling errors in the parameter space but dependent on only the acceleration information error and the control time delay due to the digital implementation. Since a control input is continuous and accurate acceleration can be obtained, the maximum of resultant disturbances, N becomes very small, thus we can design a new SMC without chattering.

Now, a stability property of the system (5) with control laws (18) -(22) will be stated in the next theorem.

Theorem 1 : Consider the robot system given by Equations (18)-(22). Assume that for some positive γ , $\|s(t_o)\| \leq \gamma$ and $\|x(t_o)\| < \gamma/\kappa$ are satisfied at the initial time $t = t_o$, and if the gain k_{x2} satisfies

$$k_{x2} \geq N - k_{x1} \cdot \delta \quad (28)$$

for given k_{x1} and δ , then the global control system is uniformly bounded (i.e. the solution X is uniformly bounded at origin in state space) for all $t \geq t_o$ except $\|s(t)\| \geq \eta_1$ where η_1 is defined by

$$\eta_1 = \sqrt{\alpha_1^2 + \beta_1^2} - \alpha_1, \quad \alpha_1 = \frac{[\delta + \frac{(k_{x2}-N)}{k_{x1}}]}{2}, \quad \beta_1 = \frac{\delta \cdot N}{k_{x1}}. \quad (29)$$

Proof

If we take $V(t) = 1/2 s^T(t)s(t)$ as a Lyapunov candidate and differentiate with respect to time, it follows

$$\begin{aligned} \frac{dV(t)}{dt} &= s^T(t) \cdot \dot{s}(t) \\ &= s^T(t) \cdot n_1(t) - s^T(t) \cdot \{k_{x1} \cdot s(t) + k_{x2} \cdot \frac{s(t)}{\|s(t)\| + \delta}\}. \end{aligned} \quad (30)$$

If we use the matrix inequality

$$\|x^T A y\| \leq \|x\| \cdot \|y\| \cdot \|A\|$$

it follows

$$\frac{dV(t)}{dt} \leq \|s(t)\| \cdot \|n_1(t)\| - k_{x1} \cdot \|s(t)\|^2 - k_{x2} \cdot \frac{\|s(t)\|^2}{\|s(t)\| + \delta}. \quad (31)$$

So that $\|n_1(t)\| \leq N$ is satisfied from the definite of N , and we can rewrite Equation (31) as

$$\begin{aligned} \frac{dV(t)}{dt} &\leq \|s(t)\| \cdot \{N - k_{x1} \cdot \|s(t)\| - k_{x2} \cdot \frac{\|s(t)\|}{\|s(t)\| + \delta}\} \\ &= -\frac{\|s(t)\| \cdot k_{x1}}{\|s(t)\| + \delta} \cdot \{\|s(t)\|^2 + 2 \cdot \alpha_1 \cdot \|s(t)\| - \beta_1\} \end{aligned} \quad (32)$$

at $t = t_1$. If the gain k_{x1} , and k_{x2} satisfy the condition (27),

$$\frac{dV(t)}{dt} < 0 \quad (33)$$

at $t \geq t_1$ until $\|s(t)\| \geq \sqrt{\alpha_1^2 + \beta_1^2} - \alpha_1$ which completes the proof of Theorem 1.

Theorem 1 guarantees the uniform bounded stability of the proposed continuous SMC (18)-(22) for robot manipulators. This controller is shown in Fig. 1, its structure is simple. The sampling time can be as small as possible so that the acceleration information is accurate enough, therefore, the maximum values N is very small. If a smaller δ is used in control algorithm (22), then the lower bound of η_1 is decreased. The lower bound η_1 can be decreased by increase of k_{x1} for given δ and N so that η_1 is sufficiently small in comparison to γ . If

the initial position error is small which is reasonable in case of the known initial state of robot manipulators, it is evident that $s(t)$ remains closer to the sliding surface $s(t) = 0$ and the trajectory error is also small, hence we may assume small γ for the desired specifications of the small trajectory errors. The feedback control (18)-(22) designed by Theorem 1 and Lemma 1 maintains the stability of the system with the prescribed performance. To design the proposed sliding mode controller, firstly the desired sliding surface defining which defines the desired error dynamics is chosen, i.e., the gains, K_v and K_p are specified. Then the gains, k_{x1} and k_{x2} , in Equation (22) are selected based in Theorem 1. The design does not need the information of maximum bound of system parameter variations or uncertainties because of the efficient on-line compensation. The proposed SMC can be designed to be accurate and robust against to parameter uncertainties.

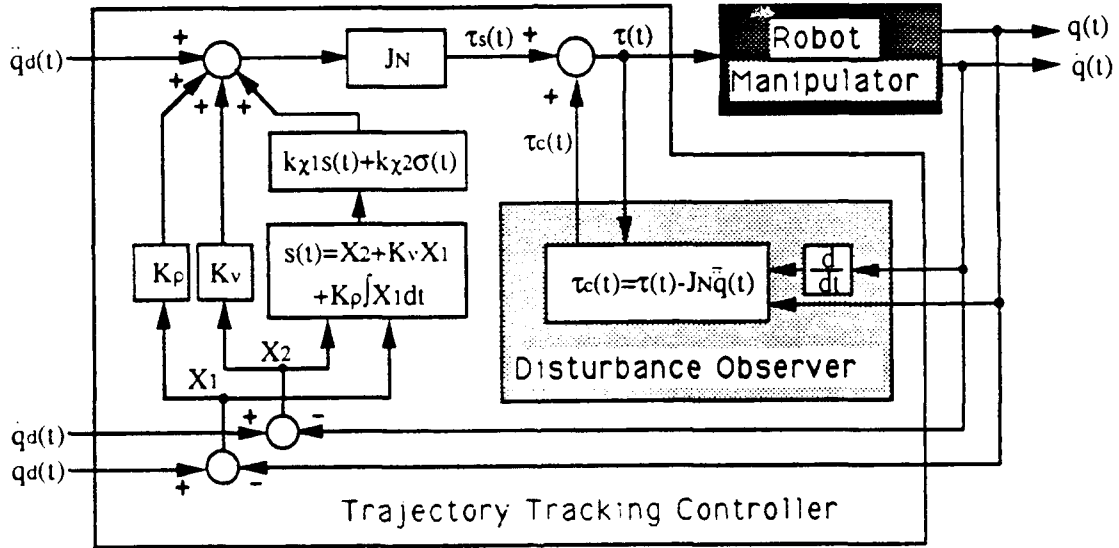


Fig. 1. The block diagram of the proposed SMC.

$$\sigma(t) = \begin{cases} \frac{s(t)}{\|s(t)\| + \delta} & \text{for Theorem 1} \\ \frac{s(t)}{|s(t)| + \delta} & \text{for Theorem 2} \end{cases}$$

4 Modified Sliding Control for Robot Manipulators

In this section, we present a modified control algorithm from the control law of the previous section so that the efficient computation is possible in the case of the digital implementation. The modification replaces $\tau_x(t)$ in Equation (22) by

$$\tau_x(t) = -J_N \cdot (k_{x1} \cdot s(t) + k_{x2} \cdot \sigma_b(t)) \quad (24)$$

where the i -th element of vector $\sigma_b(t) = [\sigma_1(t), \sigma_2(t), \dots, \sigma_n(t)]^T$ is defined by

$$\sigma_i(t) = \frac{s_i(t)}{|s_i(t)| + \delta}. \quad (35)$$

If we apply the control given by Equations (18)-(21) and (34) to the system (1), then

$$\dot{s}(t) = n_1(t) - k_{x2} \cdot s(t) - k_{x1} \cdot \sigma_b(t) \quad (36)$$

where the disturbance vector $n_1(t)$ is also given by Equation (25). Let's denote its i -th element by $n_{1i}(t)$. For some positive numbers γ, ε and $\xi > 0$, let the constant N_∞ be defined as follows

$$N_\infty = \sup_{t,z,w,y} \left\{ \sum_i \|n_{1i}(\Delta \tilde{q}(t), \Delta \tau(t), \hat{\phi})\|; q(t) \in E(\varepsilon_1; q_d(t) \wedge \dot{q}(t) \in E(\varepsilon_2; \dot{q}(t))) \right\} \quad (37)$$

where the neighborhood set is defined as

$$E(\rho; \nu) = \{w \in R^n; \|w - \nu\|_\infty \leq \rho\} \quad (38)$$

for any scalar ρ and vector $\nu \in R^n$, where $\|\cdot\|_\infty$ is the infinity norm of the vector which is defined as the maximum absolute value of its components.

Theorem 2 : Consider the robot system (5) with controls given by (18)-(21) and (34). Assume that for some $\gamma > 0$, $\|s(t_0)\|_\infty \geq \gamma$ and $\|x(t_0)\|_\infty \leq \gamma/\kappa$ is satisfied at the initial time t_0 . If the gain k_{x2} satisfies

$$k_{x2} \leq N_\infty - \delta \cdot k_{x1} \quad (39)$$

for given k_{x1} and δ , then the global control system is uniformly bounded (i.e. the solution X is uniformly bounded at origin in state space) for all $t \geq t_0$ until $\|s(t)\| \leq \eta_2$ where η_2 is defined by

$$\eta_2 = \sqrt{\alpha_2^2 + \beta_2^2} - \alpha_2, \quad \alpha_1 = \frac{\delta + \{\frac{k_{x2} - N_\infty}{k_{x1}}\}}{2}, \quad \beta = \frac{\delta \cdot N_\infty}{k_{x1}}. \quad (40)$$

Proof

If we take $V(t) = 1/2 s^T(t)s(t)$ as a Lyapunov candidate and differentiate it with respect to time, it follows

$$\begin{aligned} \frac{dV(t)}{dt} &= s^T(t) \cdot \dot{s}(t) \\ &= \{s^T \cdot n_1(t) - s^T(t) \cdot (k_{x1} \cdot s(t) + k_{x2} \cdot s(t) \cdot \sigma_b(t))\}. \end{aligned} \quad (41)$$

We can rewrite the above equation as

$$\frac{dV(t)}{dt} = \left\{ \sum_i s_i(t) \cdot n_{1i}(t) - \sum_i k_{x1} \cdot s_i(t)^2 - \sum_j k_{x2} \cdot \frac{s_j(t)^2}{|s_j(t)| + \delta} \right\}. \quad (42)$$

So it follows

$$\frac{dV(t)}{dt} \leq \left\{ \sum_i |s_i(t)| \cdot |n_{1i}(t)| - \sum_i k_{x1} \cdot s_i(t)^2 - \sum_j k_{x2} \cdot \frac{s_j(t)^2}{|s_j(t)| + \delta} \right\}. \quad (43)$$

From the definition of N_∞ and norm inequality of $\|s(t)\| \geq \|s(t)\|_\infty$, we can rewrite the equation (43) as

$$\begin{aligned} \frac{dV(t)}{dt} &\leq \|s(t)\|_\infty \{N_\infty - k_{x1} \cdot \|s(t)\|_\infty - k_{x2} \cdot \frac{\|s(t)\|_\infty}{\|s(t)\|_\infty + \delta}\} \\ &= -\frac{k_{x1} \cdot \|s(t)\|_\infty}{\|s(t)\|_\infty + \delta} \cdot \{\|s(t)\|_\infty^2 + 2 \cdot \alpha_2 \cdot \|s(t)\|_2 - \beta_2\}. \end{aligned} \quad (44)$$

If the gain k_{x1} , and k_{x2} satisfy the condition (39),

$$\frac{dV(t)}{dt} < 0 \quad (45)$$

at $t \geq t_o$, until $\|s(t)\| \geq \sqrt{\alpha_2^2 + \beta_2^2} - \alpha_2$, the proof of Theorem 2 is completed.

The desired performance of the modified controller for robot manipulators can be obtained by using Theorem 2. The feedback sliding mode control (18)-(21) and (34) maintain the uniformly bounded stability of the system as far as the condition (39) is satisfied, but Theorem 2 is only a sufficient condition for stability. Under the same conditions, N_∞ defined in Equation (37) is always larger than N defined in Equation (26). Thus as shown in Equations (39) and (40), for the same trajectory error the gains of the modified controller are designed to be larger than those of the SMC defined in section 3. Under the same conditions for the both controls, the error of the modified version be smaller than that of the original SMC because of $\|s(t)\| \geq |s_i(t)|$.

5 Numerical Simulation

The numerical simulations are performed for the purpose that is to show accurate and robust trajectory tracking property of the proposed algorithms compared with the CTM. The SCARA-type two degree-of-freedom manipulator dynamics [1] used for simulation are as follows

$$\begin{aligned} \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} &= l_2 \cdot \begin{bmatrix} \frac{1}{3}m_1 + \frac{4}{3}m_2 + m_2C_2 & \frac{1}{3}m_2 + \frac{1}{2}m_2C_2 \\ \frac{1}{3}m_2 + \frac{1}{2}m_2C_2 & \frac{1}{3}m_2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \\ &+ l_2 \cdot \begin{bmatrix} -\frac{1}{2}m_2S_2\dot{q}_2^2 - m_2S_2\dot{q}_1\dot{q}_2 \\ \frac{1}{2}m_2S_2\dot{q}_1^2 \end{bmatrix} + l \cdot \begin{bmatrix} \frac{1}{2}m_1gC_1 + \frac{1}{2}m_2gC_{12} + m_2gC_1 \\ \frac{1}{2}m_2gC_{12} \end{bmatrix} \end{aligned} \quad (46)$$

where C_i , S_i and C_{ij} imply $\cos(q_i)$, $\sin(q_i)$ and $\cos(q_i + q_j)$, respectively. The parameters are $m_1 = m_2 = 0.782[\text{kg}]$, $l = 0.23[\text{m}]$ and $g = 9.8[\text{m/sec}^2]$. The reasonable modeling error of robot dynamics is produced for each mass and length. Moreover, unknown payload is reflected to m_2 . To test the robustness to modeling uncertainties and unknown payload, simulation studies are carried for the CTM and two proposed SMCs under three case conditions, i.e., (i): no modeling error and no unknown payload, (ii): 10 [%] modeling error and 0.5 [kg] unknown payload, and (iii): 10 [%] modeling error and 1 [kg] unknown payload. The desired trajectory for each link was

$$q_d(t) = \{q_i + \frac{(q_f - q_i)}{2} \cdot t - \frac{(q_f - q_i) \cdot \sin(\pi t)}{2\pi}\} \cdot \frac{180}{\pi} \text{ [degree]} \quad (47)$$

where the initial position $q_i = [0.0 \ 0.0]^T$ [degree], and the final position $q_f = [60. \ -60.]^T$ [degree]. We take the execution time of the trajectory tracking as 2 seconds. The necessary time for the computation of model (2) was selected to be 2 [msec], and this computation time is considered as a time delay of input for the simulation of the CTM. The gain $k_p = 400 \cdot I$, $k_v = 40 \cdot I$ were used in the sliding surface (6) so that the ideal sliding dynamics have its eigenvalues to be assigned to -20. The corresponding constants defined in (9), K and κ become 14.7 and 10. Using the results of Lemma 1, the trajectory error, X_1 , and its derivative, X_2 , are bounded as $\epsilon_1 = 1.47 \cdot \gamma$ and $\epsilon_2 = 592 \cdot \gamma$ for a given γ of the constraint to the sliding surface. The smaller bound of the sliding surface, γ , the smaller trajectory error and its derivative. For 0.01 [degree] maximum tracking error, γ was selected to be 0.0068.

Now, in order to obtain the above prescribed tracking performance, the proposed SMCs are designed to make the sliding surface be bounded by γ . Assuming the initial position of the robot manipulator is known, the conditions, $\|X(t_0)\| < \gamma/\kappa$ and $\|s(t_0)\| < \gamma$ are satisfied at initial time $t = t_0$. The controller gains k_{x1} and k_{x2} were selected to be 50 and 10 which satisfy the conditions (39) and (45) for given $\delta = 0.05$ and N or N_∞ so that η_1 and η_2 are sufficiently small with respect to the selected γ by Theorem 1 and Theorem 2. For comparison of the three case conditions, the N and N_∞ are summarized in Table 1. As the payload increases, the value N and N_∞ increased. As a results the lower bound of k_{x2} becomes larger such that the stability of the system is maintained. The simulation block diagram of proposed algorithms are shown in Fig. 1. The proposed trajectory tracking controller is consisted of the modified equivalent control, $\bar{\tau}_{eq}(t)$ (21), feedback of the sliding surface, $\tau_x(t)$ (22) and disturbance observer, $\tau_c(t)$ (19), terms.

The gains $K_v = 40 \cdot I$, $K_p = 400 \cdot I$ were used in the CTM algorithm. The simulation results of the CTM are shown in Fig. 2. The tracking errors for the CTM are shown in Fig. 2-1 (a) for link one and (b) for link two, and its corresponding control inputs are shown in Fig. 2-2 (a) for link one and in Fig. 2-2 (b) for link two. As the payload increases, the root-mean-square (rms) error is rapidly increased from 0.0073 [degree] to 3.6801 [degree] using the CTM algorithm. Thus the CTM need to be added the robust algorithm against model uncertainties and unknown payloads.

The results of the proposed SMC simulations are shown in Fig. 3 for Theorem 1 and in Fig. 4 for Theorem 2. The tracking errors of the proposed SMC are presented in Fig. 3-1 and Fig. 4-1. The tracking errors by Theorem 2 are smaller than those by Theorem 1 because the norm of the sliding surface in (22) is greater than the infinity norm of the sliding surface in (35). The correspondent continuous control inputs and sliding surfaces are depicted in Fig. 3-2 and Fig. 3-3 for Theorem 1 and in Fig. 4-2 and Fig. 4-3 for Theorem 2. The simulation results in view of maximum and rms values are summarized in Table 1. Using the proposed algorithm, the resultant error maintains nearly unchanged Especially, note that using the proposed algorithm the tracking error is not increased so much as the increase of the payload error, for the case Theorem 1 and Theorem 2.

From the simulations, we have found that the proposed two algorithms provides a better performance than the CTM algorithm with respect to the tracking errors. Moreover, the tracking errors of the proposed algorithms satisfy the prescribed specifications as shown in Fig. 3-1 and Fig. 4-1 subject to parameter and payload uncertainties.

Table I. Numerical comparison between CTM and proposed SMC

Method	CASE	Error (<i>max</i>) [<i>degree</i>]	Error (<i>rms</i>) [<i>degree</i>]	Control (<i>max</i>) [Nm]	Control (<i>rms</i>) [Nm]	Surface (<i>max</i>)	Surface (<i>rms</i>)	<i>N</i>	<i>N_u</i>
CTM	1	0.0066	0.0073	3.6438	3.7569				
	2	3.6375	3.6801	5.7958	5.8300				
	3	6.3872	6.4713	8.0360	8.0453				
SMC	THM 1	1	0.0004	0.0005	3.6438	3.7569	0.0015	0.0016	0.3281
		2	0.0008	0.0010	5.3968	5.6014	0.0031	0.0034	0.4862
		3	0.0011	0.0016	7.1498	7.4476	0.0041	0.0044	0.6462
	THM 2	1	0.0002	0.0002	3.6439	3.7569	0.0008	0.0008	0.4681
		2	0.0004	0.0005	5.3968	5.6014	0.0015	0.0016	0.6911
		3	0.0005	0.0006	7.1498	7.4476	0.0020	0.0021	0.9153

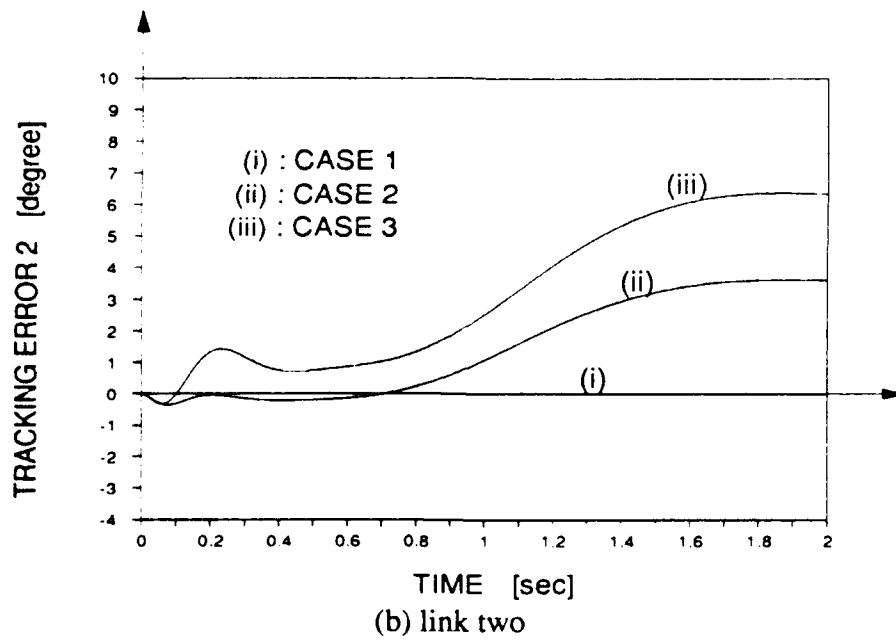
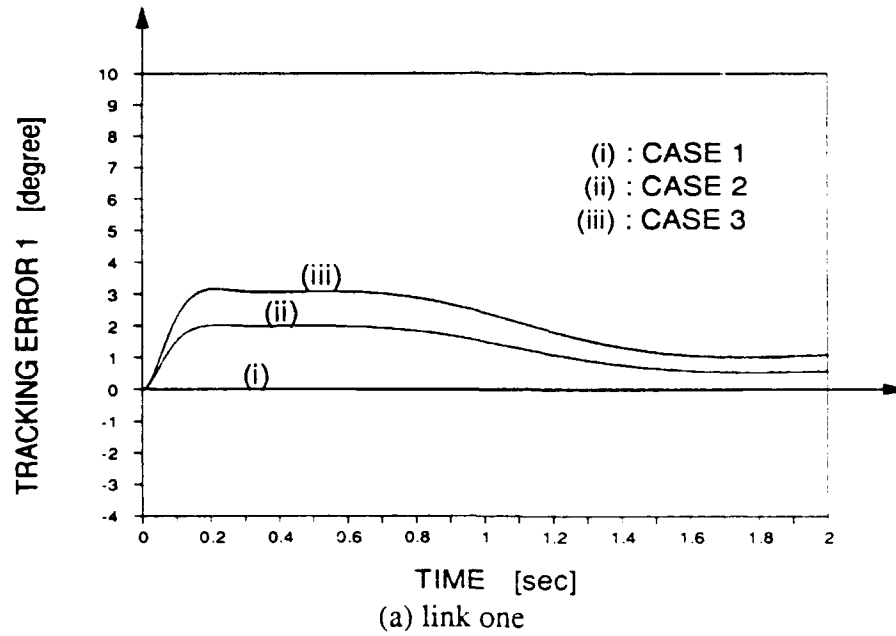


Fig. 2-1 Tracking errors for CTM algorithm

- (i) case 1 : no model error and no unknown payload
- (ii) case 2 : 10 [%] model error and 0.5 [kg] unknown payload
- (iii) case 3 : 10 [%] model error and 1.0 [kg] unknown payload

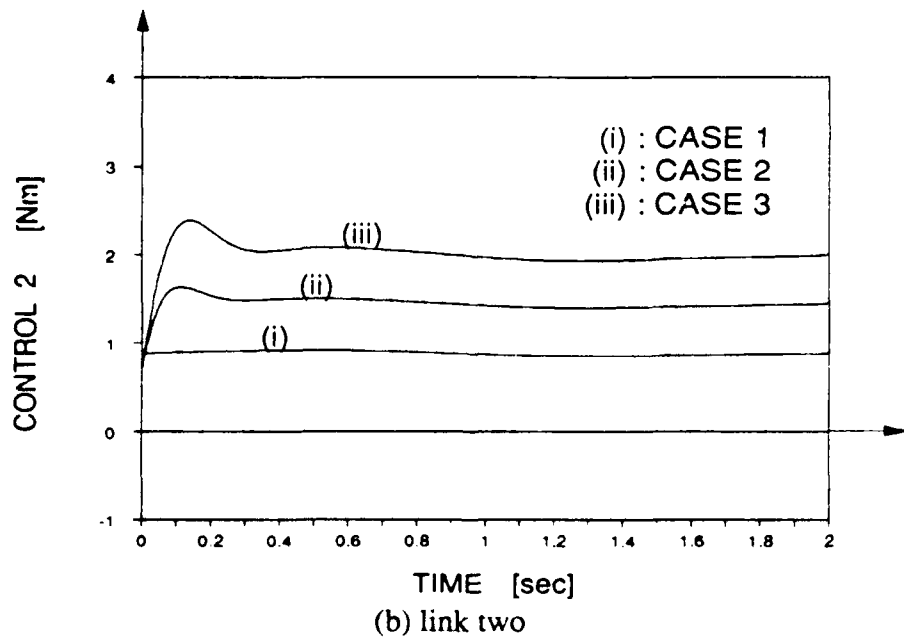
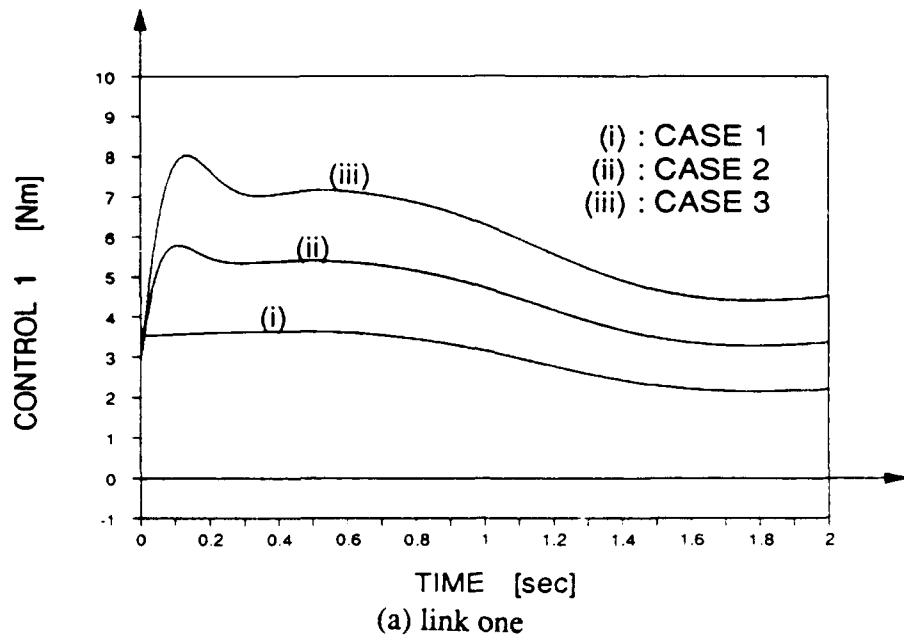


Fig. 2-2 Corresponding control inputs by CTM algorithm

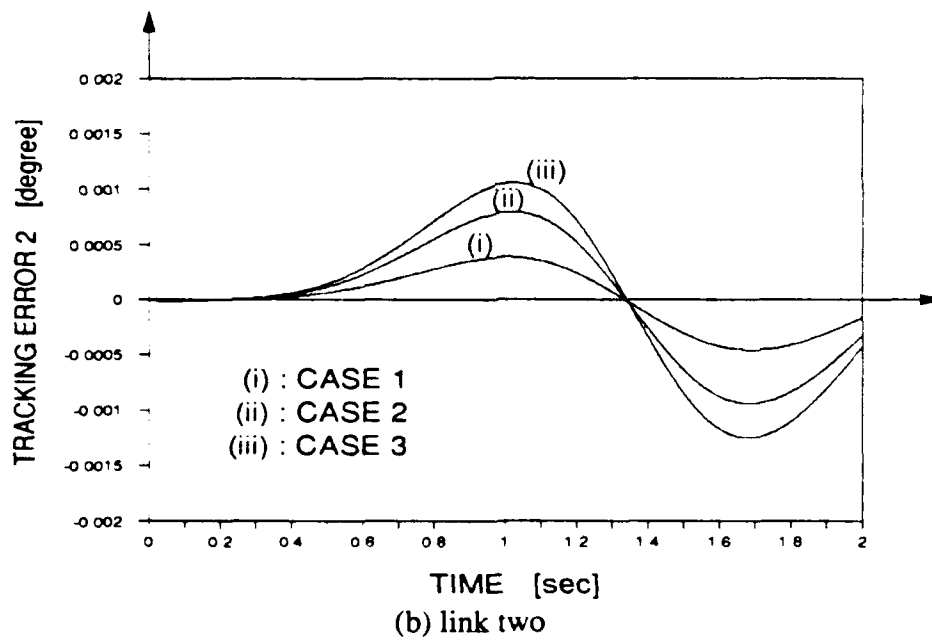
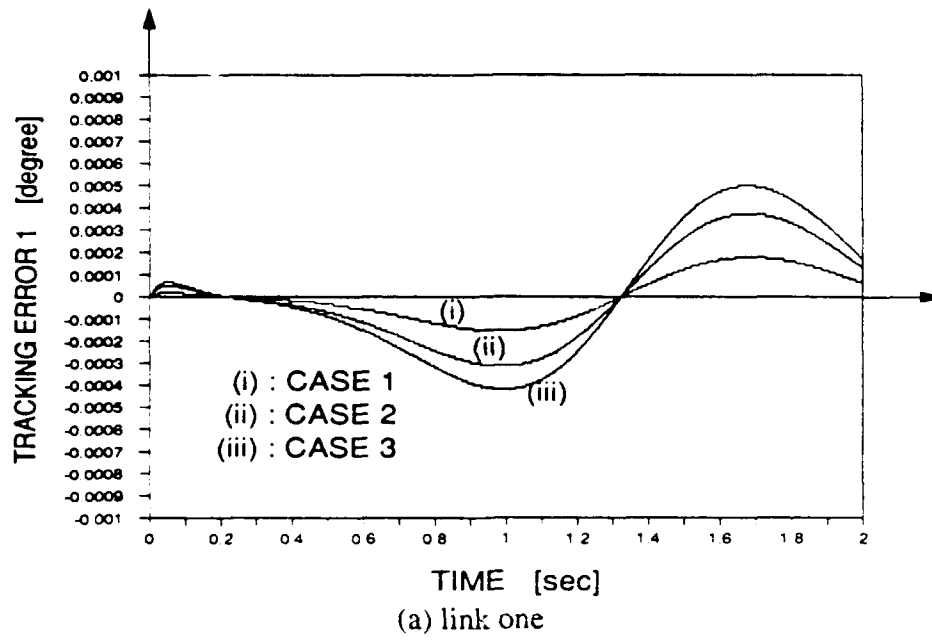


Fig. 3-1 Tracking error for Theorem 1 of proposed SMC

- (i) case 1 : no model error and no unknown payload
- (ii) case 2 : 10 [%] model error and 0.5 [kg] unknown payload
- (iii) case 3 : 10 [%] model error and 1.0 [kg] unknown payload

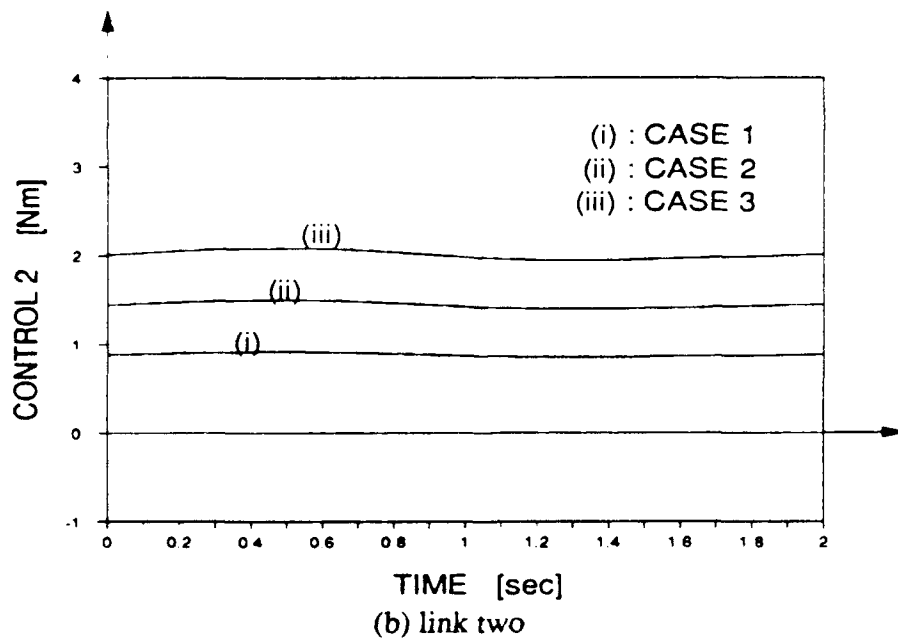
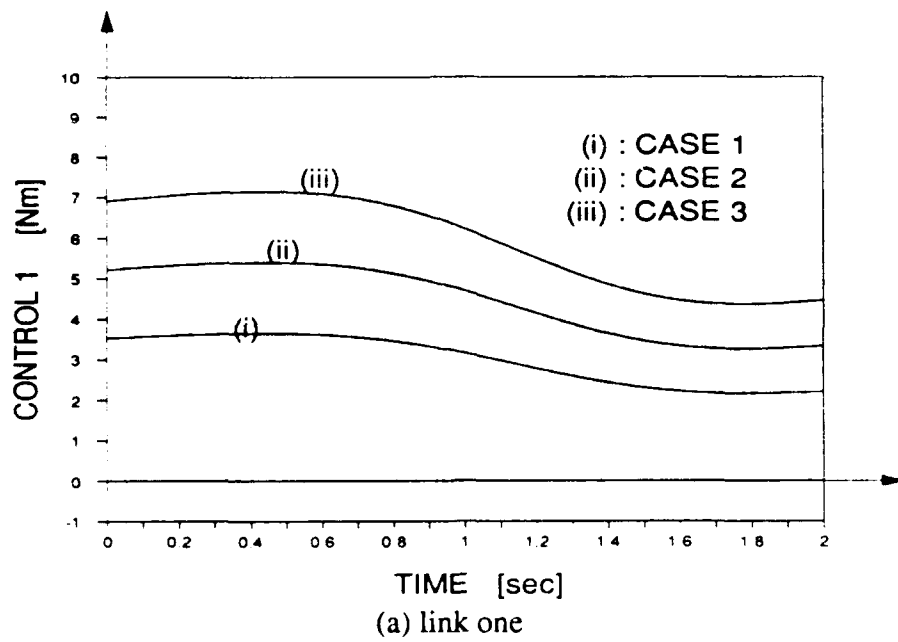


Fig. 3-2 Corresponding control inputs by Theorem 1 of proposed SMC

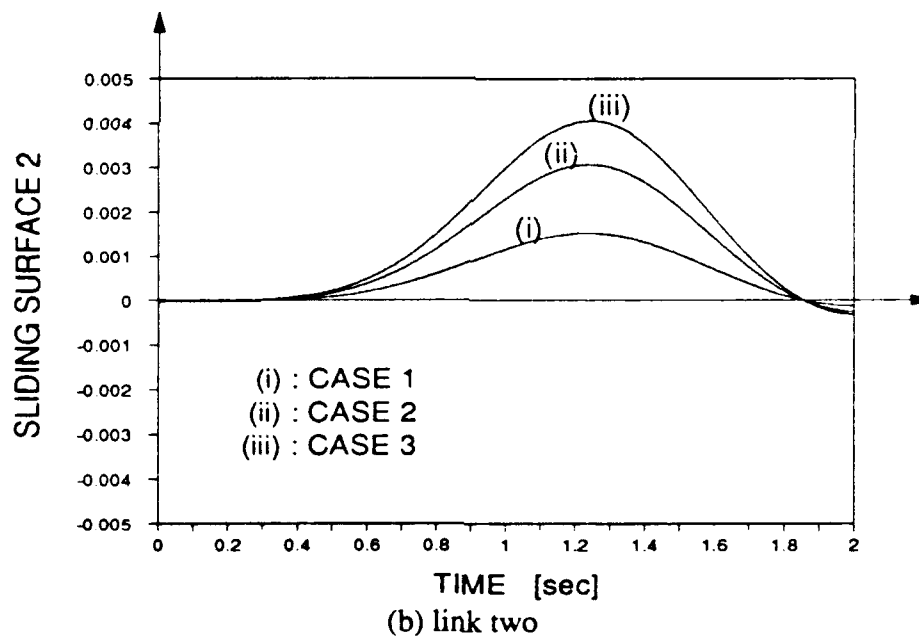
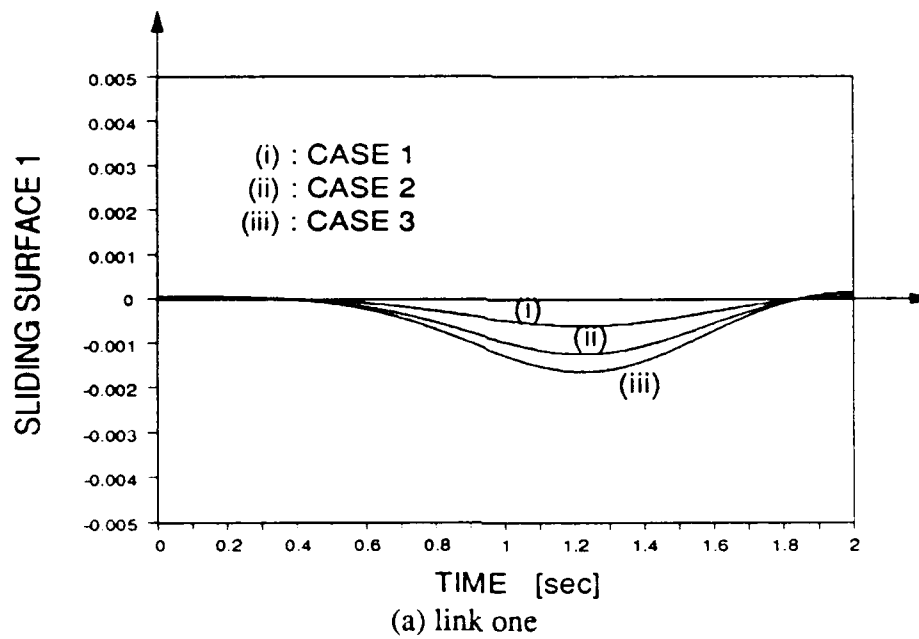
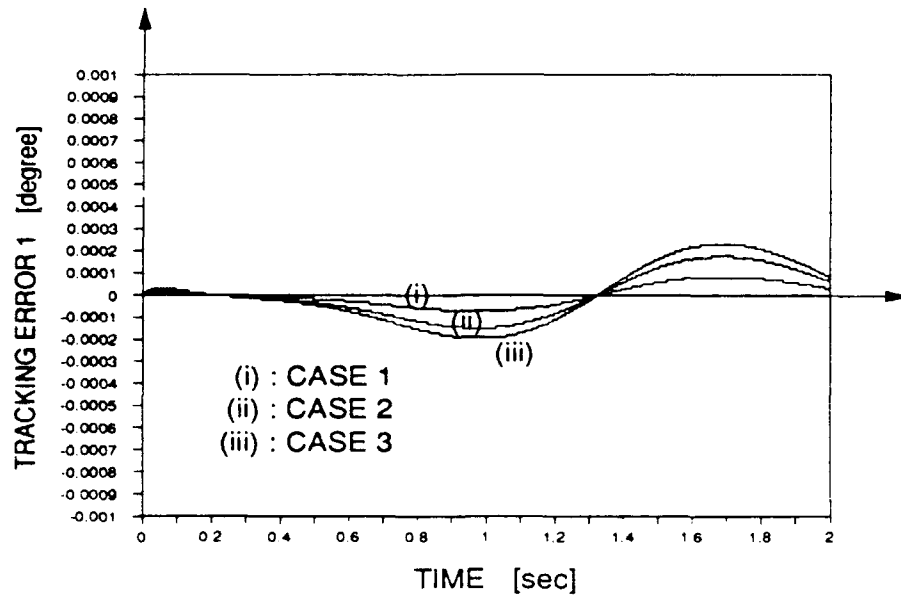
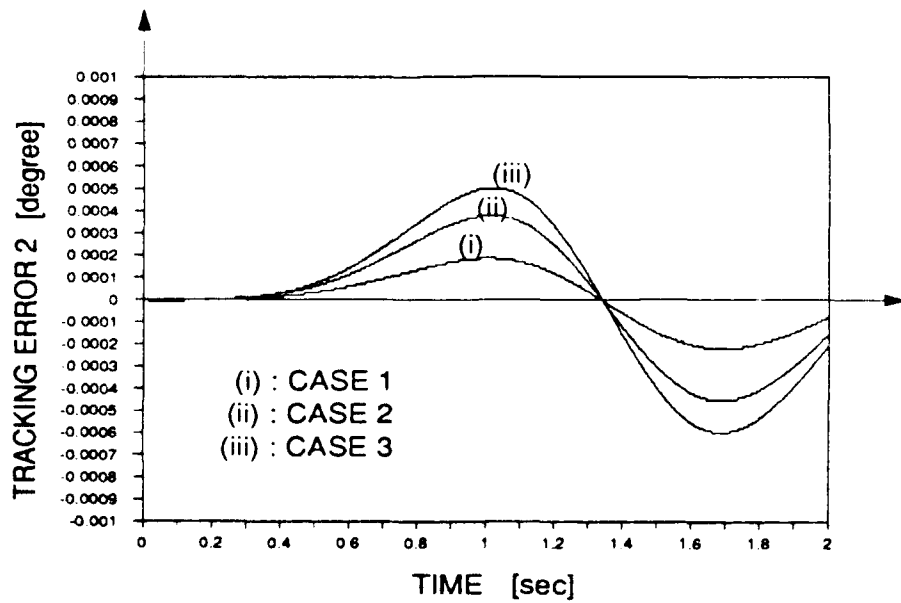


Fig. 3-3 Corresponding sliding surfaces for Theorem 1 of proposed SMC



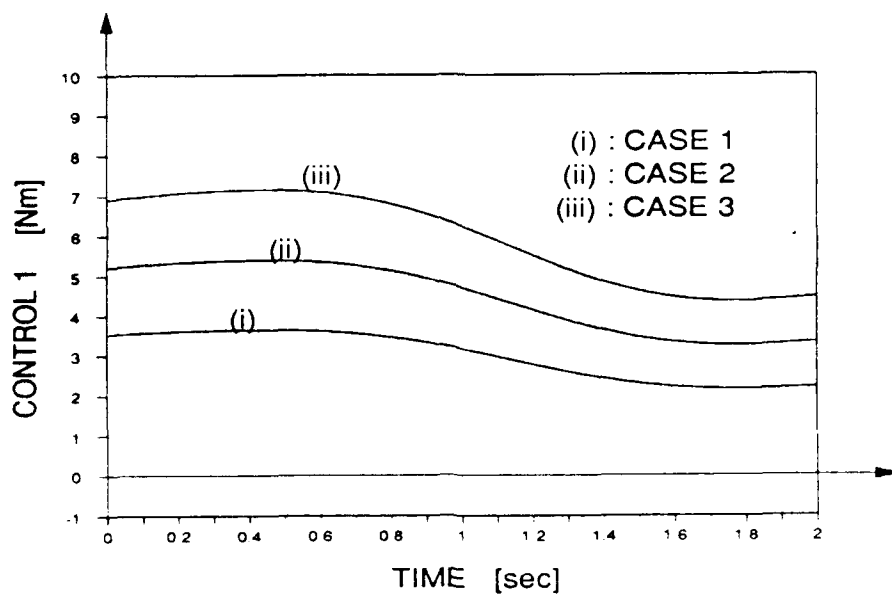
(a) link one



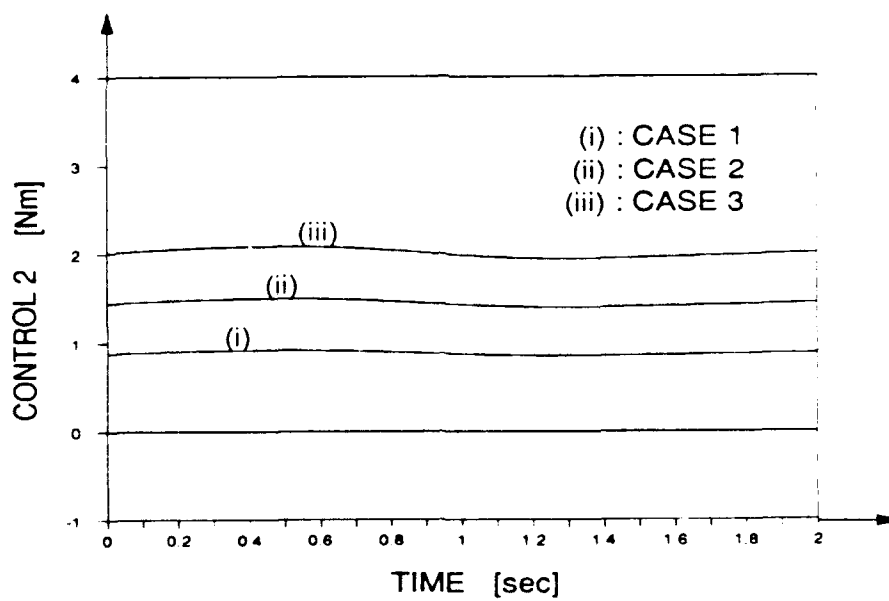
(b) link two

Fig. 4-1 Tracking errors for Theorem 2 of proposed SMC

- (i) case 1 : no model error and no unknown payload
- (ii) case 2 : 10 [%] model error and 0.5 [kg] unknown payload
- (iii) case 3 : 10 [%] model error and 1.0 [kg] unknown payload

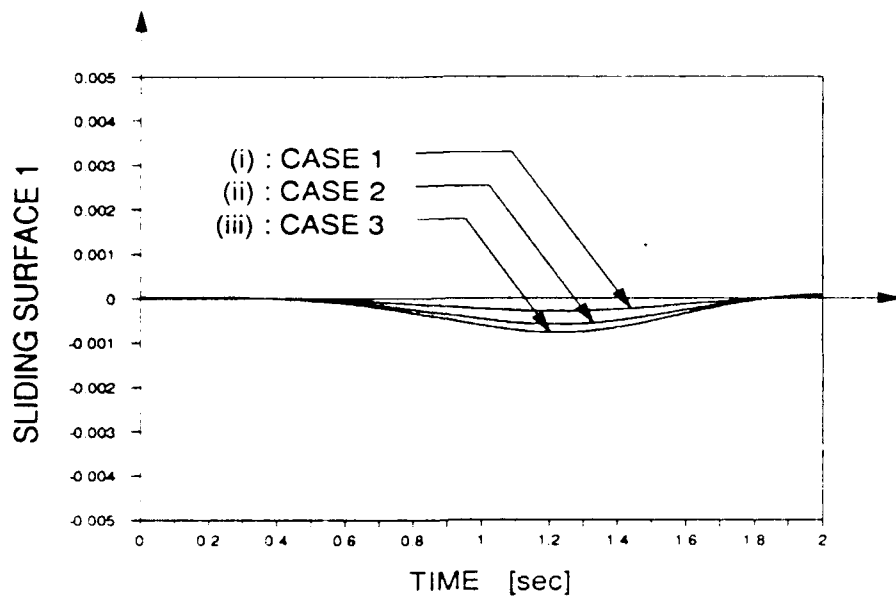


(a) link one

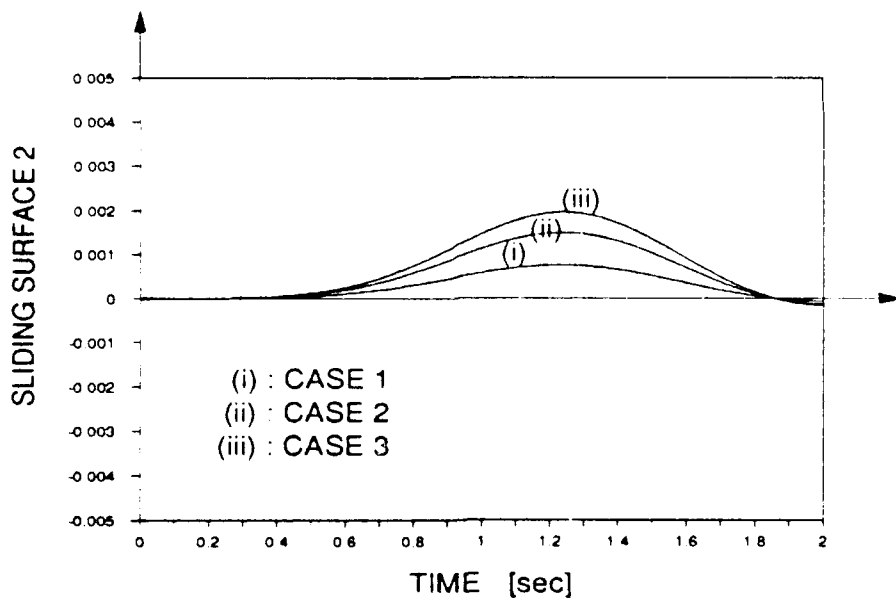


(b) link two

Fig. 4-2 Corresponding control inputs by Theorem 2 of proposed SMC



(a) link one



(b) link two

Fig. 4-3 Corresponding sliding surface for Theorem 2 of proposed SMC

6 Conclusions

In this paper, continuous sliding control algorithms have been proposed for accurate and robust tracking control of robot manipulator and their stability properties are analyzed in detail. As a preliminary work, the relationship between the maximum bound of the tracking error and that of the sliding surface is presented in Lemma 1. The continuous sliding mode control algorithm with disturbance observer is proposed and its uniform bounded stability property is proven in Theorem 1. To improve efficiency, a modification to the proposed algorithm has been preceded and its stability property is presented in Theorem 2. Under these control algorithms, the tracking errors can be reduced to the prescribed range without chattering problems based on the above stability analysis.

The simulation results have shown that the proposed algorithms are very efficient for the trajectory tracking and robust to the modeling inaccuracy and unknown payloads.

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